Observational Astronomy: Lecture 1

Ross McLure

Room U17, Royal Observatory; rjm@roe.ac.uk

1 Coordinate systems

To locate an object on the sky, astronomers need to define two–dimensional coordinate systems within the celestial sphere. These coordinate systems follow the same principal as the definition of the geographic coordinates latitude and longitude on the earth. They have: (i) a principal axis about which the systems rotates [the earth's rotation axis]; (ii) perpendicular to this, a great circle which is the principal reference circle along which one coordinate is measured [the equator, around which longitude is defined, for geographic coordinates]; and (iii) an infinite number of secondary reference circles that are great circles perpendicular to the principal reference circle, which meet at the poles of the principal axis [lines of constant longitude for geographic coordinates], and along which the second coordinate [latitude] is measured.

1.1 The Equatorial coordinate system

The most important coordinate system in astronomy is the **Equatorial** coordinate system, which is universally used for extra-galactic objects, and often used for galactic targets as well. It uses the rotational plane of the earth as its reference. The coordinates are **Right Ascension (RA)**, defined around that rotational plane, and usually measured in hours (from 0 to 24, increasing to the east), and **Declination (Dec)** ranging from 0 at the equator to ± 90 degrees at the north and south poles (see Fig. 1). While the equator naturally provides the zero point for Declination, it is necessary to define the zero point of Right Ascension (i.e. 0h direction) in some way. The definition involves the **ecliptic**, which is the apparent path of the Sun's motion on the celestial sphere as seen from Earth. The ecliptic plane is tilted 23.5° with respect to the plane of the celestial equator since the Earth's spin axis is tilted 23.5° with respect to its orbit around the sun. The celestial equator therefore intersects the ecliptic at two points, which are called *equinoxes*. The zero point of RA (i.e. RA= 0h) is defined as the direction of the Sun at the northern vernal (spring) equinox (around 21 March).

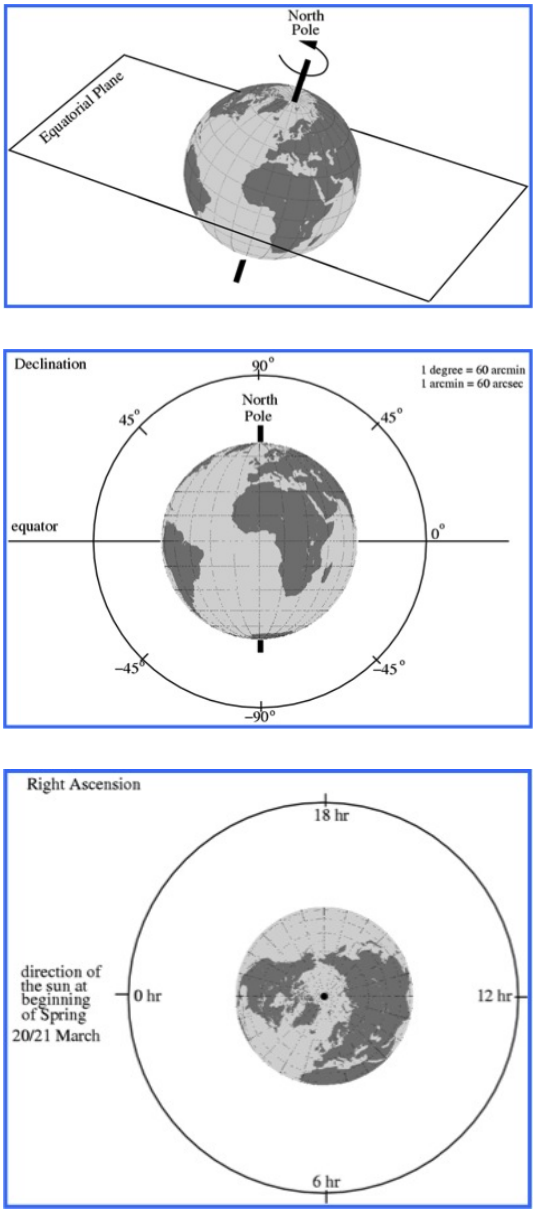

Figure 1: Equatorial coordinates - right ascension and declination.

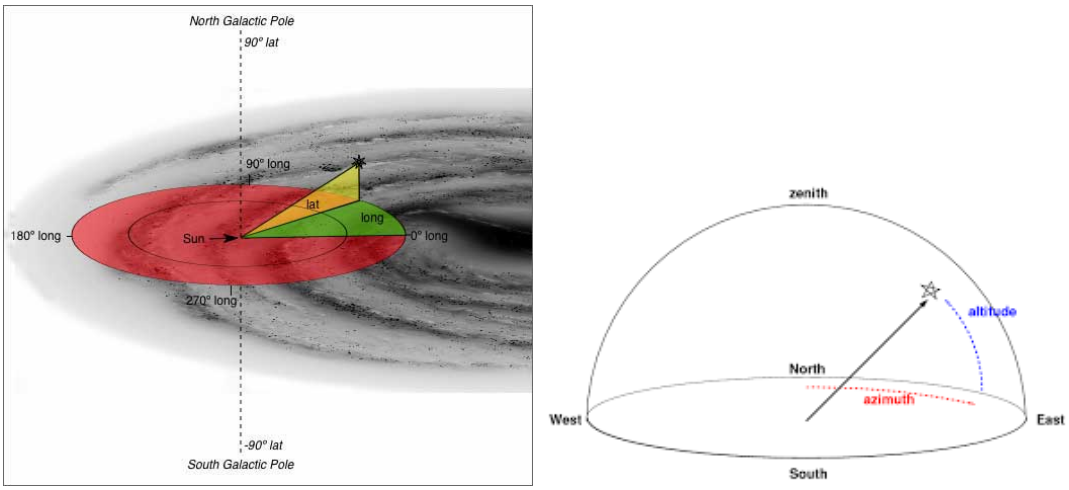

Figure 2: LHS: galactic coordinate system. RHS: horizon coordinate system (alt-az).

1.2 Alternative coordinate systems

Although the equatorial system is the primary coordinate system used throughout astronomy, other coordinate systems are used in specific situations. For example, when studying objects within our own galaxy it is sometimes helpful to refer to their positions in *galactic coordinates*. In this system, the plane of the Milky Way acts as the reference plane, and coordinates are measured in galactic latitude and galactic longitude (see Fig. 2). Another system used in astronomy is the horizon coordinate system, whereby the position of an object is defined by its *altitude* and *azimuth*. The horizon system (often referred to as *alt-az*) is primarily used for pointing telescopes and is a *local* coordinate system, in that the positions of objects on the sky change depending on your location on the Earth. In this context, equatorial and galactic coordinates are described as *global*, because they are independent of your location on Earth.

2 Angular Separation

In observational astronomy it is often necessary to calculate the angular separation between two points on the sky, based on their equatorial coordinates. If we have two objects which have the same RA, but different values of Dec, then the angular separation (in degrees) is simply:

$$\text{Ang Sep} = \Delta Dec \tag{1}$$

In contrast, if you have two objects with the same Dec, but different RAs, the angular separation is given by:

$$\text{Ang Sep} = 15 \times \cos(Dec) \times \Delta RA \tag{2}$$

where the factor of 15 converts ΔRA from units of (hours, minutes, seconds) into degrees (remember 1 hour = 15 degrees, 24 hours= 360 degrees). The requirement for the $\cos(Dec)$ factor is illustrated

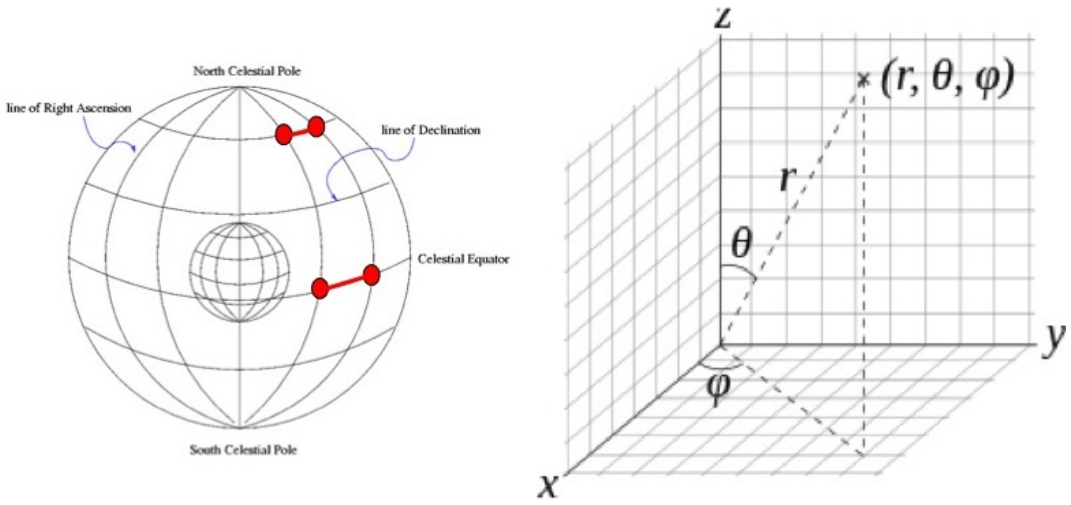

Figure 3: Left-hand side: illustration of the need for the $\cos(Dec)$ factor when calculating the angular separation of two points on the celestial sphere with constant ΔRA. Right-hand side: the relationship between Cartesian and spherical coordinates.

by Fig. 3, from which it can be seen that lines of constant RA merge at the celestial poles. Therefore, the angular separation of two points with the same ΔRA is maximal at the celestial equator (i.e. $\cos(Dec) = 1$) and shrinks to zero at the poles (i.e. $\cos(Dec) = 0$).

2.1 Small angle separation

In general, we want to be able to calculate the angular separation of two arbitrary points on the sky, with different values of both RA and Dec. Due to the fact that we are dealing with the celestial sphere, this is slightly more complicated than in flat Euclidian space. However, if the separation of the two points is small enough that the curvature of the surface is negligible, then it is still possible to use the standard approach of a right-angled triangle:

$$\text{Ang Sep} = \sqrt{\Delta Dec^2 + (15\cos(Dec)\Delta RA)^2} \tag{3}$$

2.2 Large angle separation

For large angular separations, it is not possible to use a right-angle triangle to calculate the separation due to the curved nature of the surface. In this situation it is easier to use spherical coordinates (r, θ, ϕ), as shown in the right-hand side of Fig. 3:

$$x = r\sin(\theta)\cos(\phi)$$
$$y = r\sin(\theta)\sin(\phi)$$
$$z = r\cos(\theta)$$

On the surface of a unit sphere (i.e. $r = 1$, see Fig. 4), the relationship between spherical and Cartesian coordinates becomes:

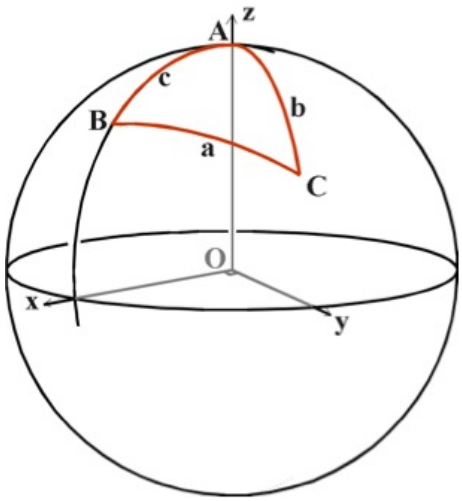

Figure 4: The geometry on the unit sphere necessary to derive the spherical cosine rule. The spherical triangle has vertex angles (A, B, C), and angular separations (a, b, c). The spherical cosine rule allows you to calculate the angular separation of two arbitrary points (B and C).

$$x = \sin(b)\cos(A)$$
$$y = \sin(b)\sin(A)$$
$$z = \cos(b)$$

where (A, B, C) are the vertex angles and (a, b, c) are the corresponding arc-lengths (or angular separations) of a spherical triangle.

Based on the geometry shown in Fig. 4, it is possible to derive the spherical cosine rule:

$$\cos(a) = \sin(b)\cos(A)\sin(c) + \cos(b)\cos(c)$$

which provides an expression for the angular separation between two points on the sphere (B, C).

Note: you are not expected to be able to derive the spherical cosine rule, but you will be expected to be able to use it to perform calculations.

3 Astrometry in Practice: measuring distances

A difficult question in astronomy is how one measures the distance to an object. The most direct way to do this is by means of *parallax*. The position of a (nearby) source is followed during the course of a year, and changes relative to the more distant sources due to the rotation of the earth around the sun (see Fig. 5). The distance corresponding to a parallax of 1 arcsec is defined to be 1 parsec (pc), which is 3.08×10^{16}m, or 3.26 light-years. For example Vega, the second brightest star in the northern sky, has a parallax of 0.123 arcsec, so its distance is $\frac{1}{0.123} = 8.13$ parsec.

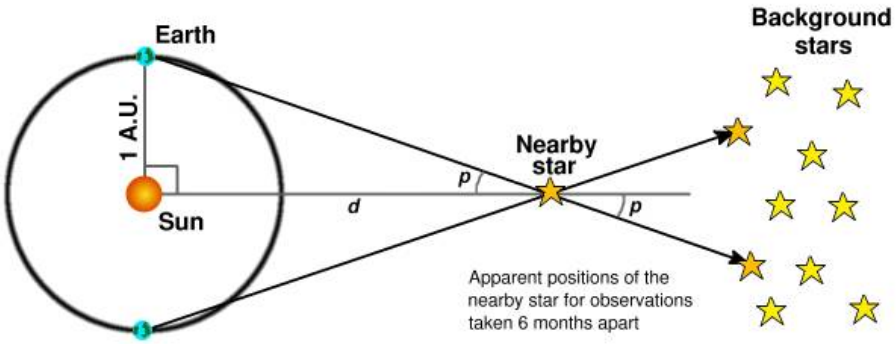

Figure 5: The concept of parallax. By measuring the position of the nearby star at two times, 6 months apart, we can measure the parallax angle p. Given the radius of the Earth's orbit around the Sun, a simple right-angled triangle provides the distance to the nearby star.

4 Solid Angle

Before discussing the quantitative measurement of light in astronomy, it is first necessary to be familiar with the concept of solid angle. Solid angle (Ω) is the two-dimensional equivalent of a normal angle, and is defined within three-dimensional space. Perhaps the easiest way to think of solid angle (see Fig. 6) is as the two-dimensional angle subtended at the centre of a sphere by an area A on the surface of the sphere. Solid angle is measured in *steradians* (basically square radians), or square degrees. The differential solid angle $\delta\Omega$ is defined as:

$$\delta\Omega = \delta A / r^2$$

which means that over the whole sphere, the solid angle is: $\Omega = 4\pi r^2/r^2 = 4\pi$ steradians and in terms of square degrees, the whole sphere is $4\pi(180/\pi)^2 = 41,253$ square degrees.

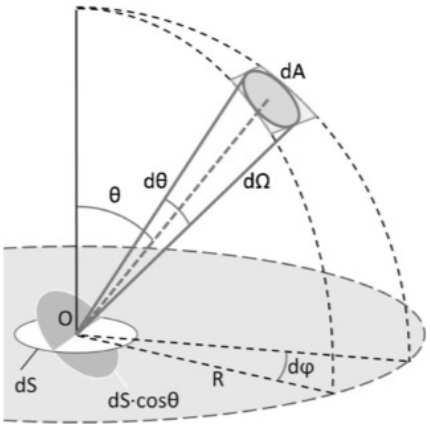

Figure 6: The geometry which provides the definition of solid angle.

5 Quantitative Measures of Light

For the quantitative description of the radiation field from astronomical objects, and its measurement, we use the following quantities:

Flux Density: this is the radiation energy received per unit time, per unit area (normal to the propagation direction of the radiation) per unit frequency (or wavelength) range. For most astronomical observations, 'per unit frequency' is used, and the flux density, f_ν, is therefore measured in units of $\mathrm{W\,m^{-2}\,Hz^{-1}}$. A commonly–used unit for measurement of flux density is the **Jansky**. $1\,\mathrm{Jansky} = 10^{-26}\mathrm{W\,m^{-2}\,Hz^{-1}}$.

Wavelength is used instead of frequency as a measure of bandwidth when there are practical reasons for doing so, for example in spectrographs which produce a nearly linear wavelength scale. We then use f_λ, measured in units of $\mathrm{W\,m^{-3}}$.

Flux: this is the integrated flux density within a given range of wavelengths or frequencies:

$$F = \int_{\nu_1}^{\nu_2} f_\nu d\nu; \qquad F = \int_{\lambda_1}^{\lambda_2} f_\lambda d\lambda; \tag{4}$$

Obviously, it can't make any physical difference to the total Flux if we decide to measure flux density per unit frequency (f_ν) or per unit wavelength (f_λ), which means that the two expressions for flux density must be related as follows (remember $c = \nu\lambda$):

$$f_\lambda = \frac{d|\nu|}{d|\lambda|} f_\nu = \frac{c}{\lambda^2} f_\nu \tag{5}$$

Specific Intensity: has units of $\mathrm{W\,m^{-2}Hz^{-1}sr^{-1}}$ and is measured in the source frame (i.e. flux density *emitted* per solid angle). Due to specific intensity being defined *per solid angle* it is conserved with distance. This is simply because $\delta\Omega = \delta A/r^2$ and $\delta A \propto r^2$.

Written as I_ν, it is related to flux density by

$$f_\nu = \int_\Omega I_\nu \cos\theta d\Omega. \tag{6}$$

where the $\cos\theta$ factor effectively quantifies the surface area normal to the direction of propogation of the incoming radiation. If the radiation is arriving from directly overhead, this means that $\cos\theta = 1$ and you receive all the radiation. However, at the other extreme, when the radiation is arriving along the horizon, $\cos\theta = 0$ and you receive none of the radiation.

Luminosity: this is the total power (radiation energy per unit time) emitted by an astrophysical source. *Monochromatic luminosity* is luminosity per wavelength or frequency unit. The luminosity is related to the flux (and monochromatic luminosity to flux density) by the distance to the source, d as:

$$L = 4\pi d^2 f; \qquad L_\nu = 4\pi d^2 f_\nu \tag{7}$$