Observational Astronomy: Lecture 2

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1 Magnitudes

In practice, we measure flux densities or brightnesses within some certain frequency (or wavelength) range. The total energy measured is then the integral of the source flux times some frequency-dependent effective filter response. This last quantity includes all the factors that modify the energy arriving at the top of the Earth's atmosphere. The main factor is the instrumental filter, but atmospheric absorption and frequency-dependent sensitivity of the detector also matter (more on this in later lectures).

At least originally, measurements of brightness in astronomy were fundamentally relative, which is what leads to the notion of magnitude. The definition of magnitude is one of astronomy's unfortunate pieces of historical baggage. Around 150 BC, Hipparcos catalogued ∼1000 visible stars in 6 categories of apparent brightness, and in 1856 Pogson suggested they be called magnitudes, with the brightest stars to the eye being 'first magnitude', and fainter stars having larger magnitude. Most human senses are logarithmic, and it turned out that a difference of 1 magnitude corresponds roughly to a factor 2.5 difference in flux density, and so the magnitude system was defined as:

$$m_1 - m_2 = -2.5 \log_{10} \left(\frac{F_1}{F_2} \right) \tag{1}$$

where m_1 and m_2 are the magnitudes and F_1 and F_2 are the fluxes of two objects measured on a given telescope. It can be seen from this equation that the magnitude system is fundamentally a relative measurement of brightness. However, we also want to measure the magnitudes of individual objects and be able to measure them on a system which everyone can agree on. To do this we need to define a suitable **zero point**.

Traditionally this is done by insisting that the magnitude of Vega (the 2nd brightest star in the northern celestial hemisphere) is zero in all filters, i.e:

$$m_1 - m_{\rm vega} = -2.5 \log_{10} \left(\frac{F_1}{F_{\rm vega}} \right) \tag{2}$$

which, if we *define* $m_{\rm vega} = 0$ in all filters, leads to:

$$m = -2.5 \log_{10} (F_1) + 2.5 \log_{10} (F_{\rm vega}) \tag{3}$$

or

$$m = -2.5 \log_{10} (F_1) + zpt \tag{4}$$

Although the zero-point will be different for every telescope/detector combination, simply by measuring the brightness of Vega with your equipment, this definition makes it possible for everyone to measure the magnitudes of individual objects and agree on the values. This process is referred to as measuring magnitudes on the same *system* (the Vega system in this case).

Unfortunately, because of the complicated spectral shape of Vega (see Fig. 1), converting between Vega magnitudes and physical fluxes is very messy (basically you need to know the physical flux of Vega at every wavelength). In recent times, it has become much more popular to use the AB magnitude system, whereby the zero point is defined to be 3631 Jy in all bands (the flux density of Vega in the middle of the optical atmospheric window).

This is possible because, although the fundamental magnitude equation (Eqn. 1) is defined in terms of Flux, it is entirely equivalent to write it in terms of *flux density* (i.e. f_ν or f_λ):

$$m = -2.5 \log_{10} (\langle f_\nu \rangle) + C \tag{5}$$

where, in the Vega magnitude system, C is:

$$2.5 \log_{10} (\langle f_\nu^{\rm vega} \rangle) \tag{6}$$

In the AB magnitude system the quantity $\langle f_\nu^{\rm vega} \rangle$, which varies significantly with frequency/wavelength, is replaced by a constant flux density of 3631 Jy. This removes the need to know the detailed spectrum of Vega and makes the conversion between magnitudes and average flux densities very straightforward, i.e.:

$$m = -2.5 \log_{10} (\langle f_\nu \rangle) + 2.5 \log_{10} (3631 \text{Jy}) \tag{7}$$

$$m = -2.5 \log_{10} (\langle f_\nu \rangle) + 8.9 \tag{8}$$

where the flux density (f_ν) is measured in Jansky (Jy) units.

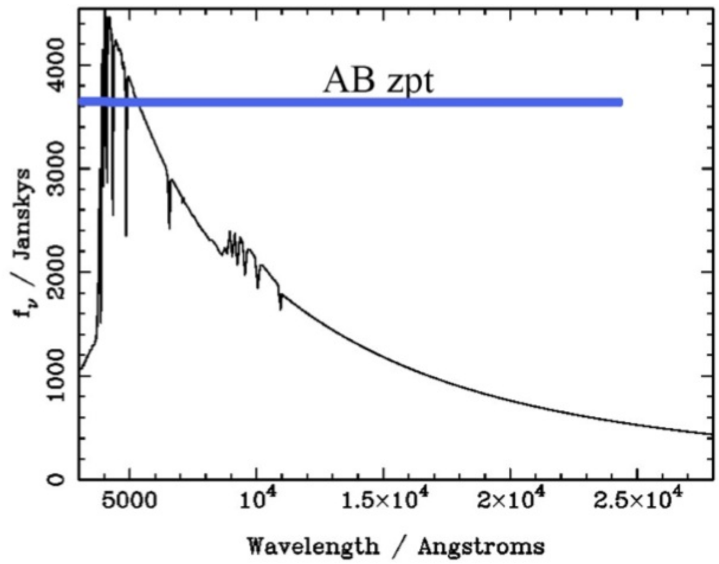

Figure 1: The spectrum of Vega (flux-density versus wavelength). The horizontal bar shows the adopted zero-point for the AB magnitude system.

2 Absolute magnitude

If you know the distance to an astronomical object, you can use the apparent magnitude to calculate the **absolute magnitude**. The absolute magnitude is the magnitude form of the intrinsic luminosity, and is defined as the apparent magnitude that would be observed if the source lay at a distance of 10 pc:

If we start with:

$$m_1 - m_2 = -2.5 \log_{10} \left(\frac{F_1}{F_2} \right) \tag{9}$$

and let m_1 be the absolute magnitude (always written as M) and m_2 be the apparent magnitude (m), then we have:

$$M - m = -2.5 \log_{10} \left(\frac{F}{f} \right) \tag{10}$$

where: $F = \frac{L}{4\pi(10\text{pc})^2}$ and $f = \frac{L}{4\pi d^2}$

$$\boxed{M = m - 5 \log_{10} \left(\frac{d}{10\text{pc}} \right)} \tag{11}$$

where d is the distance to the source (measured in parsecs). The quantity $m - M = 5 \log_{10}(d/10\text{pc})$ is also called the *distance modulus*.

3 Filters

It would be ideal to measure the flux density of astronomical objects using filters which are maybe 1Å wide (or narrower). However, it is usually the case that astronomical objects are sufficiently faint that you have to observe them in much wider filters. In Fig. 2 you can see the transmission profiles of common optical, nearIR and midIR filters, which are typically > 1000Å wide. When you measure the apparent magnitude of an object in one of these filters, you are basically measuring the average flux density of the object over the wavelength range of the filter:

$$m = -2.5 \log_{10}(\langle f_\lambda \rangle) + C \tag{12}$$

with

$$\langle f_\lambda \rangle = \frac{\int f_\lambda \lambda R_\lambda \delta\lambda}{\int \lambda R_\lambda \delta\lambda} \tag{13}$$

where R_λ is the filter transmission function and the λ weighting accounts for the fact that we are typically talking about CCD detectors, which are fundamentally *photon counting* devices (more on this in later lectures). Equation 13 can be derived by realising that because the energy of a photon is $E_p = hc/\lambda$, the number of photons detected (N_p) will be:

$$N_\mathrm{p} \propto \lambda E \tag{14}$$

where E is the total energy detected. This means that:

$$N_\mathrm{p} \propto \int f_\lambda \lambda R_\lambda \delta\lambda \tag{15}$$

which we can also write as:

$$N_\mathrm{p} \propto \langle f_\lambda \rangle \int \lambda R_\lambda \delta\lambda \tag{16}$$

If we equate Eqns. 15 & 16 we arrive at the expression for $\langle f_\lambda \rangle$ in Eqn. 13.

Having derived an expression for the average flux density measured through the filter, it is often useful to discuss the so-called *effective wavelength* of the filter, which is defined such that:

$$\langle f_\lambda \rangle \sim f_\lambda(\lambda_\mathrm{eff}) \tag{17}$$

One definition of the effective wavelength is called the *pivot wavelength*, which is defined as:

$$\lambda_\mathrm{eff}^2 = \frac{\int \lambda R_\lambda \delta\lambda}{\int R_\lambda \frac{\delta\lambda}{\lambda}} \tag{18}$$

As expected, for most filters the pivot wavelength is very close to the mean wavelength of the filter (see Table 1 for the effective wavelengths and widths of common filters). A full derivation of the pivot wavelength definition is given at the end of the notes.

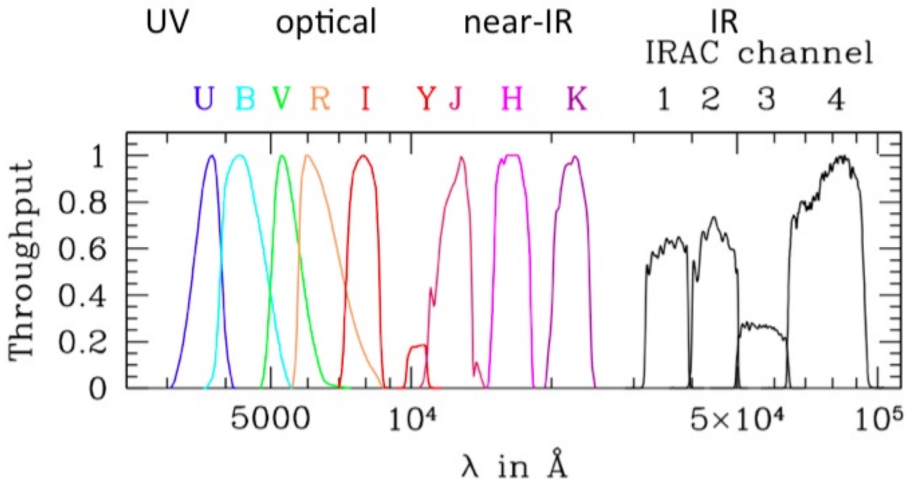

Figure 2: The transmission profiles of common filters in the optical-midIR, which range from ~ 1000Å wide in the optical to ≥ 6000Å wide in the midIR.

Filter	$\lambda_{\rm eff}$/nm	$\Delta\lambda$/nm
U	360	65
B	430	100
V	550	80
R	680	95
I	900	230
J	1220	150
H	1630	170
K	2190	190
L	3450	280

Table 1: Parameters for common filter systems. The first column lists the filter name, the second column lists the effective wavelength and the third column lists the filter width.

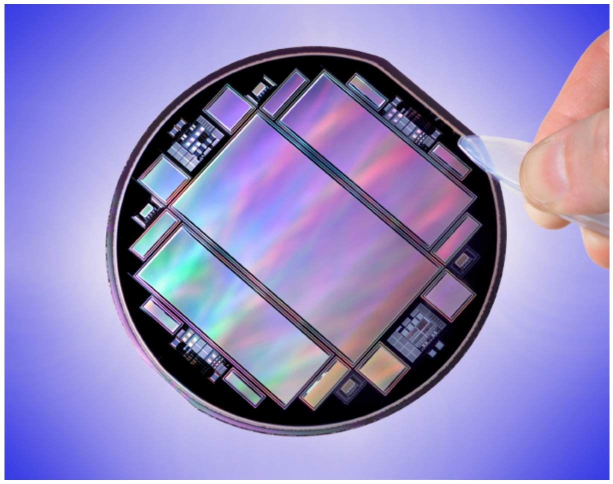

Figure 3: CCD detectors are manufactured on silicon wafers. The image shows a silicon wafer with a diameter of 10cm which contains a number of large and small CCD arrays. The largest array in the centre has 2048×4096 pixels, i.e. a 8 mega-pixel device. The image is taken from: `http://www-ccd.lbl.gov/`

4 CCD detectors

Over the last thirty years, charge-couple devices (CCDs) have completely replaced photography as the detector of choice for optical astronomy. A CCD detector (see Fig. 3) consists of a two-dimensional array of pixels made from photo-sensitive semi-conductor material, which generates electrons when it absorbes incoming photons (via the photo-electric effect). They are designed such that during an exposure (or *integration*) the electrons are stored within the pixels, ready to be counted, or *read out*, at the end of the integration. The final output is a 2D array of data numbers (i.e. a digital image), in which the data number associated with each pixel is directly proportional to the number of photons that were incident upon it.

CCD detectors have completely replaced photographic film for a number of different reasons:

1. They are much more sensitive than photographic film, with quantum efficiencies (QE) approaching 100%. The QE is the ratio of the detected photons to the number of incident photons. In comparison, the QE of film is only a few percent.

2. The response of CCDs is linear, i.e. the number of electrons collected is directly proportional to the number of photons received. Definitely not true for film.

3. CCDs are electronic and automatically produce digital images.

4. CCDs are small, robust and easy to manufacture.

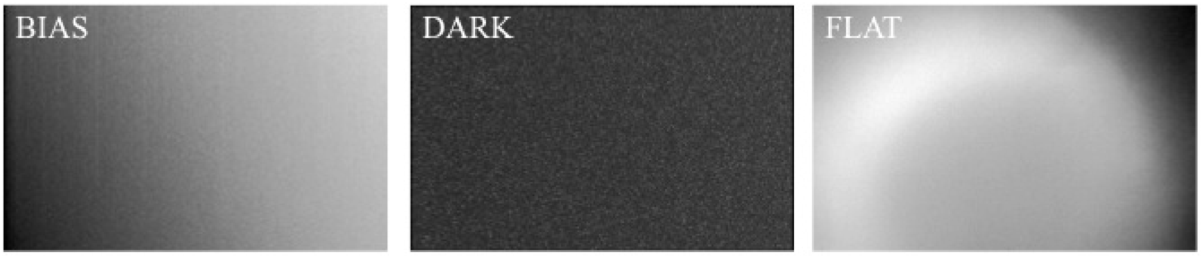

Figure 4: Examples of the three main calibration frames needed to reduce CCD imaging data. From left-to-right there is a bias frame, a dark frame and a flat-field frame. The bias frame provides a measurement of the counts in each pixel introduced by the read-out electronics. The dark frame provides a measurement of how many thermally generated electrons have be accumulated over the course of an exposure/integration. The flat-field frame provides a measurement of the relative sensitivity of the CCD pixels. See text for a discussion of how these calibration frames are employed.

5 CCD data reduction

We will discuss the physics of how CCD detectors actually *work* in a later lecture, but for now we will concentrate on how to calibrate CCD images so that they can be used for scientific analysis.

Unfortunately, the images that are initially generated by CCDs are not ready to be used for scienctific analysis. The reason for this is that these *raw* images carry with them a number of different *instrumental signatures*. The process of calibrating the images to remove these instrumental signatures is referred to as *data reduction*. In this section the different calibration steps which are necessary to convert raw CCD images into science-ready images are discussed.

5.1 Calibration images

To effectively reduce CCD imaging it is necessary to obtain a number of different calibration images:

1. **BIAS FRAME:** even using the shortest possible exposure and with the shutter closed, CCD pixels don't register zero counts because the electronics are set-up such that read-noise never makes the signal go negative. This "bias" needs to be subtracted from every image (see Fig. 5), using a bias frame generated by taking a $t_{\rm exp} = 0$ exposure with the shutter closed. A *master bias* is often constructed from a stack of bias frames in order to increase the signal-to-noise ratio (S/N).

2. **DARK FRAME:** even when CCD detectors are heavily cooled, thermally generated electrons are also trapped in the CCD pixels in addition to the desired photo-generated electrons. The thermally generated electrons are referred to as the *dark current*. To correct for this a dark frame exposure is taken with the shutter closed and the same exposure time as your science image ($t_{\rm exp} = t_{\rm sci}$, shutter closed). Again, usual practice is to stack a number of dark frames together to make a *master dark*.

3. **FLAT FIELD:** even in the best CCD detectors, the sensitivity of the individual pixels is not entirely uniform due to variations in intrinsic sensitivity, dust/dirt and non-uniform illumination of the detector (vignetting). This effect is corrected by creating a "flat field", which is an exposure of a uniform light source, normalised such that the mean pixel value is unity. Dividing the science images by the normalised flat field will therefore correct for the differences in pixel-to-pixel sensitivity. As with the bias and dark frames, it is usual practice to take a number of flat field frames and stack them together to produce a *master flat* with higher S/N.

5.2 Reduction procedure

Once you are armed with the necessary calibration images, you can then reduce a single exposure read-out from a CCD detector following the steps illustrated in Fig. 5.

The first step is to subtract the bias frame from the raw science image to remove the bias signature. The second step is to subtract the dark frame (itself bias subtracted) in order to remove the dark current. The third step is to divide the science image by the normalized flat field. At this stage you have a fully reduced science exposure. If you have already achieved your desired S/N ratio, then you are finished. However, typically you would repeat this process for a number of exposures of the target and stack them together to produce the final science image ready for analysis.

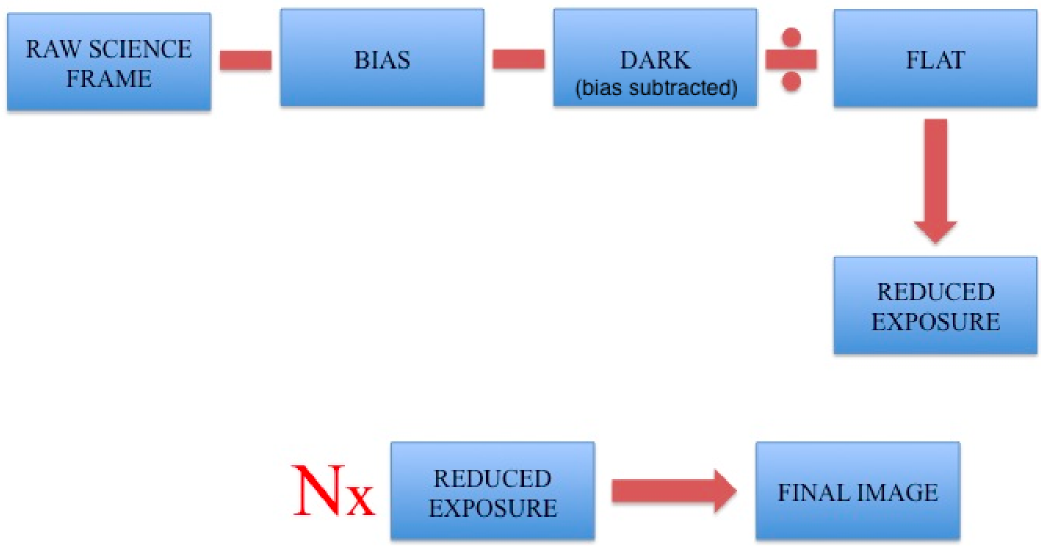

Figure 5: A schematic diagram illustrating the main steps in reducing optical CCD imaging data.

The process illustrated in Fig. 5 includes all of the necessary steps to reduce a raw CCD image. However, in reality, it is common to dispense with either the bias subtraction, or the dark subtraction, depending on the characteristics of the CCD detector. For example, if the bias signal is not a dominant source of noise, it is quite common to simply subtract off a master dark frame. This is possible because the dark frames themselves contain a measurement of the bias signal. Alternatively, if you have a CCD detector with ultra-low dark current, it is common to simply subtract off a master bias frame. Dividing through by the normalised flat-field frame is always necessary.

6 Derivation of pivot wavelength (not examinable)

As discussed in Section 3, when you measure the apparent magnitude of an object you are basically measuring the average flux density ($\langle f_\lambda \rangle$) over the wavelength range of your chosen filter. It is useful to define the *effective wavelength* of a filter ($\lambda_{\rm eff}$), such that:

$$\langle f_\lambda \rangle \sim f_\lambda(\lambda_{\rm eff}) \tag{19}$$

A rigorous definition of the effective wavelength of a filter can be derived as follows. As shown in Section 3, the mean flux density measured over the filter is:

$$\langle f_\lambda \rangle = \frac{\int f_\lambda \lambda R_\lambda \delta\lambda}{\int \lambda R_\lambda \delta\lambda} \tag{20}$$

which we can also write in terms of flux density per unit frequency:

$$\langle f_\nu \rangle = \frac{\int \frac{f_\nu}{\nu} R_\nu \delta\nu}{\int \frac{R_\nu}{\nu} \delta\nu} \tag{21}$$

or, using $f_\nu = \frac{\lambda^2}{c} f_\lambda$ and $\frac{\delta\nu}{\nu} = \frac{\delta\lambda}{\lambda}$, we can say:

$$\langle f_\nu \rangle = \frac{\int f_\lambda \lambda R_\lambda \delta\lambda}{c \int R_\lambda \frac{\delta\lambda}{\lambda}} \tag{22}$$

if we then require that:

$$\langle f_\nu \rangle = \frac{\lambda_{\rm eff}^2}{c} \langle f_\lambda \rangle \tag{23}$$

it follows that:

$$\lambda_{\rm eff}^2 = \frac{\int \lambda R_\lambda \delta\lambda}{\int R_\lambda \frac{\delta\lambda}{\lambda}} \tag{24}$$

This definition of the effective wavelength of a filter is know as the **pivot wavelength**. Unless a filter has a shape which is very different from a so-called *tophat*, the pivot wavelength is very close to the mean wavelength of the filter.